Debye screening and Meissner effect in a three-flavor color superconductor

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I compute the gluon self-energy in a color superconductor with three flavors of massless quarks, where condensation of Cooper pairs breaks the color and flavor $SU(3)_c \times U(3)_V \times U(3)_A$ symmetry of QCD to the diagonal subgroup $SU(3)_{c+V}$. At zero temperature, all eight electric gluons obtain a Debye screening mass, and all eight magnetic gluons a Meissner mass. The Debye as well as the Meissner masses are found to be equal for the different gluon colors. These masses determine the coefficients of the kinetic terms in the effective theory for the low-energy degrees of freedom. Their values agree with those obtained by Son and Stephanov.

I. INTRODUCTION AND CONCLUSIONS

The presence of attractive interactions in a degenerate fermionic system destabilizes the Fermi surface and leads to the formation of Cooper pairs — the system becomes a superconductor [1]. When increasing the quark density in cold quark matter, asymptotic freedom implies that single-gluon exchange becomes the dominant interaction between quarks. Single-gluon exchange is attractive in the color-antitriplet channel, and therefore leads to color superconductivity in cold, dense quark matter [2].

Considerable activity has been recently generated [3–40] by the observation that the zero-temperature color-superconducting gap ϕ_0 can be as large as 100 MeV [3,7]. Gaps of this magnitude may have important consequences for the physics of nuclear collisions. The critical temperature for the onset of color superconductivity, T_c, is (to leading order in the strong coupling constant g) related to ϕ_0 in the same way as in BCS theory, $T_c \simeq 0.57\, \phi_0$ [15]. Thus, for $\phi_0 \sim 100$ MeV, it cannot be excluded that color-superconducting quark matter could be created in nuclear collisions in the GSI–SIS or BNL–AGS energy range.

In order to compute the color-superconducting gap, one commonly solves a gap equation [1]. For theories with point-like four-fermion interactions, $\phi_0 \sim \mu \exp(-c_{4\mathrm{F}}/G^2)$, where μ is the quark chemical potential, $c_{4\mathrm{F}}$ is a constant, and G^2 is the four-fermion coupling strength [3–5,7,8,11,20,34]. On the other hand, in QCD, $\phi_0 \sim \mu \exp(-c_{\mathrm{QCD}}/g)$, where c_{QCD} is another constant [10,12,15,21,23–25,27,29,30]. In weak coupling, there are then three different energy scales, $\phi_0 \ll m_g \ll \mu$, where m_g is the gluon mass. At $T = 0$ and for N_f flavors of massless quarks [41],

$$m_g^2 \equiv \frac{N_f}{3}\, \frac{g^2 \mu^2}{2\pi^2} \; . \tag{1}$$

The value of c_{QCD} depends on the form of the gluon propagator in the cold, dense quark medium. If one takes the gluon propagator in the standard “hard dense loop“ (HDL) approximation [41], one obtains $c_{\mathrm{QCD}} = 3\,\pi^2/\sqrt{2}$ [25]. In this approximation, the quarks inside the HDL's are assumed to be in the normal, and not the color-superconducting phase. Consequently, an important question that has to be addressed is how color superconductivity influences the propagation of gluons and whether this could change c_{QCD}. In a recent work [19], I have derived a general expression for the gluon self-energy in a two-flavor color superconductor, and explicitly computed the self-energy in the static limit, $p_0 = 0$, for gluon momenta $p \equiv |\mathbf{p}| \to 0$, and for $p \gg \phi_0$.

In a two-flavor color superconductor, condensation of Cooper pairs in a channel of total spin $J = 0$ breaks $SU(3)_c$ to $SU(2)_c$. Then, the three gluons corresponding to the generators of the unbroken $SU(2)_c$ subgroup are expected to remain massless, while the other five should attain a mass through the Anderson–Higgs mechanism. An explicit computation of the gluon self-energy to one-loop order in perturbation theory confirms this qualitative expectation, but quantitatively reveals some surprising details [19]. At $T=0$, the three gluons of the unbroken $SU(2)_c$ attain no Meissner mass, but also no Debye mass. This means that static, homogeneous color-electric fields of the unbroken $SU(2)_c$ subgroup are not screened. Furthermore, the Debye and Meissner masses of the remaining five gluons are not identical: four gluons have a Debye mass $\sqrt{3/2}\, m_g$ and a Meissner mass $m_g/\sqrt{2}$, while the last one has a Debye mass $\sqrt{3}\, m_g$, like in the non-superconducting phase, but a Meissner mass $m_g/\sqrt{3}$.

On the other hand, in a three-flavor color superconductor the color and flavor $SU(3)_c \times U(3)_V \times U(3)_A$ symmetry is broken to the diagonal subgroup $SU(3)_{c+V}$. This locks color and flavor rotations [4]. From the 18 Goldstone bosons resulting from symmetry breaking, eight get "eaten" by the gluons, which consequently become massive. The purpose of this paper is to complement the results of [19] for the two-flavor case with the computation of the Debye and Meissner masses of these gluons in the three-flavor case.

It is worthwhile mentioning that here, as well as in [19], the terms "Debye mass" and "Meissner mass" refer exclusively to the screening of *color*-electric and *color*-magnetic fields. The "ordinary" (electro-)magnetic Meissner effect was studied in [6,36]. Similar to the mixing of weak and electromagnetic gauge bosons in the standard model, the electromagnetic field mixes with the eighth gluon to form a modified photon, under which the color-superconducting condensate is electrically neutral. The mass of the modified eighth gluon becomes slightly larger than that of the other seven. However, the mixing angle as well as this increase in mass is determined by the ratio of electromagnetic and strong coupling constants and consequently quite small. Therefore, effects from electromagnetism will be neglected throughout the following.

There is another reason why it is important to know the values for the Debye and Meissner mass. The relevant degrees of freedom in the color-flavor locked phase at energy scales much smaller than the gap, ϕ_0, are the remaining 10 Goldstone bosons resulting from the breaking of color and flavor symmetries. Apart from an additional Goldstone boson arising from breaking $U(1)_V$, these bosons are analogous to the pseudoscalar mesons which result from chiral symmetry breaking in the QCD vacuum [9,17]. Consequently, the dynamics of these 10 Goldstone bosons is described by an effective theory which resembles the Lagrangian of the nonlinear sigma model, describing the dynamics of the chiral fields in the QCD vacuum [26,31]. To lowest order, this Lagrangian contains only kinetic terms,

$$\mathcal{L}_{\mathrm{nl}\Sigma}^{\mathrm{kin}} = \frac{f_\pi^2}{4} \mathrm{Tr}\left(\partial_0 \Sigma^\dagger \partial_0 \Sigma - v_\pi^2 \boldsymbol{\nabla}\Sigma^\dagger \cdot \boldsymbol{\nabla}\Sigma\right) + 12\, f_{\eta'}^2 \left[(\partial_0\theta)^2 - v_{\eta'}^2 \left(\boldsymbol{\nabla}\theta\right)^2\right] + 12\, f_H^2 \left[(\partial_0\varphi)^2 - v_H^2 \left(\boldsymbol{\nabla}\varphi\right)^2\right] \ . \quad (2)$$

Here, $\Sigma \equiv \exp(i\,\lambda^a/f_\pi)$, where π^a, $a=1,\ldots,8$, are the fields corresponding to the meson octet in the QCD vacuum. These are the pions, the kaons, and the η meson, which are the Goldstone bosons resulting from spontaneous breaking of the $SU(3)_A$ symmetry. λ^a are the Gell-Mann matrices. θ is the field corresponding to the meson singlet in the QCD vacuum, *i.e.*, the η' meson, which is the Goldstone boson resulting from breaking $U(1)_A$ spontaneously. (In the QCD vacuum, $U(1)_A$ is also broken explicitly by instantons. At the quark densities relevant for the effective theory in the color-flavor locked phase, however, instantons play no longer any significant role [13].) Finally, φ is the Goldstone mode resulting from breaking $U(1)_V$. This field has no analogon in the QCD vacuum.

The coefficients of the kinetic terms are determined by the "decay constants" f_π, $f_{\eta'}$, and f_H. The presence of a medium breaks Lorentz invariance, and the coefficients of the time-like and space-like terms may differ, *i.e.*, the velocities v_π^2, $v_{\eta'}^2$, and v_H^2 are not necessarily equal to one. These velocities enter the dispersion relation of the Goldstone bosons as $\epsilon_i^2(k) = v_i^2\, k^2 + m_i^2$, $i=\pi,\, \eta',\, H$.

The decay constants as well as the velocities can be computed by matching the effective theory to the underlying microscopic theory. A convenient way to do this was proposed by Son and Stephanov [26]. First, they observed that, by minimally gauging the model (2), for instance the pseudoscalar decay constant f_π is related to the Debye mass of the gluons, m_D, via

$$f_\pi \equiv m_D/g \ , \quad (3)$$

and the velocity of the pseudoscalar mesons is determined from the ratio of Debye and Meissner masses,

$$v_\pi \equiv m_M/m_D \ . \quad (4)$$

The problem is thus reduced to computing these masses in the underlying theory. In this case, one has two choices. First, one may use QCD in the color-superconducting ground state. This theory has quasiparticle as well as quasi-antiparticle excitations, and is valid at all energy scales. Second, one may start from the effective theory proposed by Hong [21]. This theory contains only quasiparticle excitations around the Fermi surface, quasi-antiparticle excitations have already been integrated out. It is valid at energy scales which are much smaller than the chemical potential, μ, but which can be larger than the gap, ϕ_0.

Son and Stephanov [26] used the latter theory to compute the Debye and Meissner masses. Their results are

$$m_D^2 = m_g^2 \, \frac{21 - 8\,\ln 2}{18} \quad , \quad m_M^2 = m_g^2 \, \frac{21 - 8\,\ln 2}{54} \; . \tag{5}$$

Consequently, the velocity of the pseudoscalar mesons is $v_\pi = 1/\sqrt{3}$. Note that the (square of the) Debye mass in the three-flavor superconductor, $m_D^2 \simeq 0.859\, m_g^2$, is reduced by a factor 3.5 as compared to its value in a normal, cold medium, $m_D^2 = 3\, m_g^2$.

The result (5) is not undisputed throughout the literature. For instance, Rho, Shuryak, Wirzba, and Zahed [30] computed the Debye and Meissner masses from the gluon self-energy in the full theory, including quasi-antiparticle excitations. They obtained [see Eqs. (A.72) and (A.75) of [30]],

$$m_D^2 = \frac{1}{2}\, m_g^2 \quad , \quad m_M^2 = \frac{5}{6}\, m_g^2 \; . \tag{6}$$

This is quite surprising, as it implies that the velocity of the pseudoscalar mesons is superluminous, $v_\pi \equiv m_M/m_D = \sqrt{5/3}$. Other results that can be found in the literature are those of Zarembo [37], which agree with Son and Stephanov's calculation. Beane, Bedaque, and Savage [33] agree with Son and Stephanov on the Debye and Meissner masses up to a factor of 2.

The second goal of this paper is to resolve this ambiguity in the literature. The Debye and Meissner masses will be computed in the full theory, *i.e.*, QCD in the color-superconducting ground state. The framework for such a computation was already established in [19]. As shown in the following section, the results are found to agree with those of Son and Stephanov, Eq. (5).

The Debye and Meissner masses are not only important for the nonlinear version (2) of the effective low-energy theory in a three-flavor color superconductor. As outlined in [19] they also determine the coefficients of the kinetic terms in the *linear* version of the effective theory,

$$\mathcal{L}^{\rm kin}_{\rm l\Sigma} = \alpha_{\rm e} \sum_{h=r,\ell} {\rm Tr}\left[(D_0 \Phi_h)^\dagger \; D^0 \Phi_h \right] + \alpha_{\rm m} \sum_{h=r,\ell} {\rm Tr}\left[(D_i \Phi_h)^\dagger \; D^i \Phi_h \right] \; . \tag{7}$$

Since there is no reason why right- and left-handed terms should differ in normalization, I assumed $\alpha_{{\rm e}\,r} \equiv \alpha_{{\rm e}\,\ell} \equiv \alpha_{\rm e}$, and similarly for the coefficient of the space-like terms, $\alpha_{\rm m}$. For color-flavor locking, the order parameter is a 3×3 matrix with expectation value $\langle \Phi_h \rangle = {\rm diag}(\phi_{0\,h}, \, \phi_{0\,h}, \, \phi_{0\,h})$ [13]. Consequently, $g^2 \alpha_{\rm e}(\phi_{0\,r}^2 + \phi_{0\,\ell}^2) \equiv m_D^2$, $g^2 \alpha_{\rm m}(\phi_{0\,r}^2 + \phi_{0\,\ell}^2) \equiv m_M^2$. Due to explicit symmetry breaking by nonzero quark masses (and, at less than asymptotically high densities, instantons), the true ground state of the color-flavor locked phase corresponds to the $J^P = 0^+$ channel where $\phi_{0\,r} \equiv -\phi_{0\,\ell} \equiv \phi_0$. Then, $\alpha_{\rm e} \equiv m_D^2/(2 g^2 \phi_0^2)$, $\alpha_{\rm m}^2 \equiv m_M^2/(2 g^2 \phi_0^2)$.

I use natural units, $\hbar = c = k_B = 1$, and work in Euclidean space-time $\mathbf{R}^4 \equiv V/T$, where V is the volume and T the temperature of the system. Nevertheless, I find it convenient to retain the Minkowski notation for 4-vectors, with a metric tensor $g^{\mu\nu} = {\rm diag}(+,-,-,-)$. For instance, the space-time coordinate vector is $x^\mu \equiv (t, \mathbf{x})$, $t \equiv -i\tau$, where τ is Euclidean time. 4-momenta are denoted as $K^\mu \equiv (k_0, \mathbf{k})$, $k_0 \equiv -i\omega_n$, where ω_n is the Matsubara frequency, $\omega_n \equiv 2n\pi T$ for bosons and $\omega_n \equiv (2n+1)\pi T$ for fermions, $n = 0, \pm 1, \pm 2, \ldots$. The absolute value of the 3-momentum $\mathbf{k}$ is denoted as $k \equiv |\mathbf{k}|$, and its direction as $\hat{\mathbf{k}} \equiv \mathbf{k}/k$.

II. EXPLICIT COMPUTATION OF DEBYE AND MEISSNER MASSES

A convenient starting point to compute the gluon self-energy in the color-flavor locked phase is Eq. (68) of [19],

$$\begin{aligned} \Pi^{\mu\nu}_{ab}(P) = \frac{1}{2}\, g^2 \, \frac{T}{V} \sum_K {\rm Tr}_{s,c,f} \Big[& \Gamma^\mu_a \, G^+(K) \, \Gamma^\nu_b \, G^+(K-P) + \bar{\Gamma}^\mu_a \, G^-(K) \, \bar{\Gamma}^\nu_b \, G^-(K-P) \\ & + \Gamma^\mu_a \, \Xi^-(K) \, \bar{\Gamma}^\nu_b \, \Xi^+(K-P) + \bar{\Gamma}^\mu_a \, \Xi^+(K) \, \Gamma^\nu_b \, \Xi^-(K-P) \Big] \; . \end{aligned} \tag{8}$$

Here, the trace is over color, flavor, and spinor space. The vertices are $\Gamma^\mu_a \equiv \gamma^\mu T_a$ and $\bar{\Gamma}^\mu_a \equiv -\gamma^\mu T_a^T$. $G^\pm$ and $\Xi^\pm$ are the diagonal and off-diagonal elements of the Nambu–Gor'kov propagator for quasiparticle excitations,

$$G^\pm \equiv \left(G_0^\pm - \Sigma^\pm\right)^{-1} \quad , \quad \Xi^\pm \equiv -G_0^\mp\, \Phi^\pm\, G^\pm \; . \tag{9}$$

$G_0^\pm(K) \equiv (\gamma \cdot K \pm \gamma_0 \mu)^{-1}$ is the propagator for massless, non-interacting quarks (charge-conjugate quarks), and $\Sigma^\pm \equiv \Phi^\mp\, G_0^\mp\, \Phi^\pm$ is the quark self-energy generated by exchanging particles or charge-conjugate particles with the condensate. In mean-field approximation, the condensate Φ^+ is computed from the gap equation discussed in [15], and Φ^- can be obtained from

$$\Phi^-(K) \equiv \gamma_0 \left[\Phi^+(K)\right]^\dagger \gamma_0 \; . \tag{10}$$

In Eq. (8), the first line corresponds to the diagram in Fig. 1(a), where only the diagonal components of the Nambu–Gor'kov propagator appear, while the second line corresponds to the diagram in Fig. 1(b), formed from the off-diagonal components. Note that, at small temperatures $T \sim \phi_0$, and in weak coupling, $\phi_0 \ll \mu$, the fermion loops of Fig. 1 constitute the dominant contribution to the gluon self-energy, since contributions from gluon (or ghost) loops are relatively suppressed by a factor $T^2/\mu^2 \sim \phi_0^2/\mu^2$ [19].

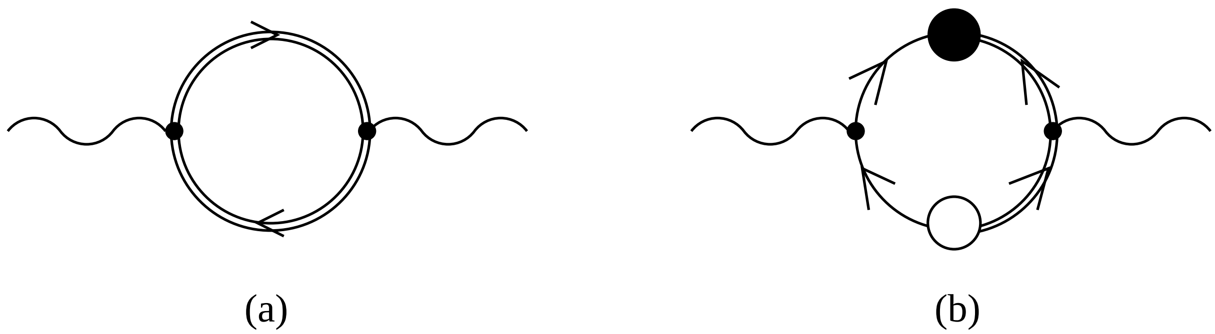

FIG. 1. The contributions from (a) the diagonal and (b) the off-diagonal components of the Nambu–Gor'kov propagator to the gluon self-energy. Double full lines stand for the full quasiparticle propagator $G^\pm$, single full lines for the free propagator $G_0^\pm$. The full blob represents Φ^-, the empty blob Φ^+. Vertices are represented by small blobs.

In the two-flavor case, the trace over color and flavor in Eq. (8) could be performed independently [19]. In the three-flavor case, due to color-flavor locking this is no longer possible. The traces over color and flavor must be performed simultaneously. The most elegant way to do this is to utilize the color-flavor space projectors introduced by Shovkovy and Wijewardhana, Eq. (7) of [24],

$$\mathcal{C}^{(1)}{}^{ij}_{rs} \equiv \frac{1}{3}\, \delta^i_r\, \delta^j_s \; , \tag{11a}$$

$$\mathcal{C}^{(2)}{}^{ij}_{rs} \equiv \frac{1}{2}\left(\delta_{rs}\, \delta^{ij} - \delta^j_r\, \delta^i_s\right) \; , \tag{11b}$$

$$\mathcal{C}^{(3)}{}^{ij}_{rs} \equiv \frac{1}{2}\left(\delta_{rs}\, \delta^{ij} + \delta^j_r\, \delta^i_s\right) - \frac{1}{3}\, \delta^i_r\, \delta^j_s \; . \tag{11c}$$

(To avoid proliferation of the symbol $\mathcal{P}$, I denote them here as $\mathcal{C}$.) All projectors are symmetric under simultaneous exchange of color, i, j, and flavor, r, s, indices. Note that $\mathcal{C}^{(1)}$ is the singlet projector $\mathbf{P_1}$ introduced by Zarembo, Eq. (3.11) in [37]. Furthermore, $\mathcal{C}^{(2)} + \mathcal{C}^{(3)} \equiv \mathbf{1} - \mathbf{P_1} \equiv \mathbf{P_8}$ is Zarembo's octet projector, Eq. (3.12) in [37].

With the projectors (11), the gap matrices $\Phi^\pm$ can be written as

$$\Phi^\pm \equiv \sum_{n=1}^{3} \mathcal{C}^{(n)}\, \Phi^\pm_n \; . \tag{12}$$

Here,

$$\Phi_1^\pm \equiv 2\left(\Phi_{\bar{\mathbf{3}}}^\pm + 2\,\Phi_{\mathbf{6}}^\pm\right) \;, \tag{13a}$$
$$\Phi_2^\pm \equiv \Phi_{\bar{\mathbf{3}}}^\pm - \Phi_{\mathbf{6}}^\pm \;, \tag{13b}$$
$$\Phi_3^\pm \equiv -\Phi_2^\pm \;, \tag{13c}$$

are gap matrices in spinor space,

$$\Phi_n^+(K) \equiv \sum_{h=r,\ell}\sum_{e=\pm} \phi_{n,h}^e(K)\,\mathcal{P}_h\,\Lambda_{\mathbf{k}}^e \quad , \quad \Phi_n^-(K) \equiv \sum_{h=r,\ell}\sum_{e=\pm}\left[\phi_{n,h}^e(K)\right]^*\,\mathcal{P}_{-h}\,\Lambda_{\mathbf{k}}^{-e} \;, \tag{14}$$

where $\mathcal{P}_{r,\ell} \equiv (1 \pm \gamma_5)/2$ are chirality projectors, $-h = \ell$ when $h = r$ and $-h = r$ when $h = \ell$, $\Lambda_{\mathbf{k}}^\pm \equiv (1 \pm \gamma_0 \boldsymbol{\gamma} \cdot \hat{\mathbf{k}})/2$ are energy projectors, and $\phi_{n,h}^e(K)$ are simple functions of 4-momentum K^μ.

In Eqs. (13), $\Phi_{\bar{\mathbf{3}}}^\pm$ is the gap matrix in the antitriplet channel and $\Phi_{\mathbf{6}}^\pm$ is the gap matrix in the sextet channel,

$$\Phi^{\pm\, ij}_{\;\; rs} \equiv \Phi_{\bar{\mathbf{3}}}^\pm \left(\delta_r^i\,\delta_s^j - \delta_s^i\,\delta_r^j\right) + \Phi_{\mathbf{6}}^\pm\left(\delta_r^i\,\delta_s^j + \delta_s^i\,\delta_r^j\right) \;. \tag{15}$$

Why condensation in the (repulsive) sextet channel is possible in the color-flavor locked phase was explained in [17]. Antitriplet and sextet gaps are related to the gap functions κ_1 and κ_2 of [4] by $\phi_{\bar{\mathbf{3}}\,\ell}^+ = -\phi_{\bar{\mathbf{3}}\,r}^+ \equiv (\kappa_1 - \kappa_2)/2$, $\phi_{\mathbf{6}\,\ell}^+ = -\phi_{\mathbf{6}\,r}^+ \equiv (\kappa_1 + \kappa_2)/2$. For future purpose, it will be convenient to define a singlet and an octet gap matrix according to

$$\Phi_{\mathbf{1}}^\pm \equiv \Phi_1^\pm \quad , \quad \Phi_{\mathbf{8}}^\pm \equiv \Phi_2^\pm \equiv -\Phi_3^\pm \;. \tag{16}$$

The quasiparticle propagators take the form

$$G^\pm(K) \equiv \sum_{n=1}^{3} \mathcal{C}^{(n)}\,G_n^\pm(K) \;, \tag{17}$$

where

$$G_n^\pm(K) = \sum_{h=r,\ell}\sum_{e=\pm} \mathcal{P}_{\pm h}\,\Lambda_{\mathbf{k}}^{\pm e}\,\frac{1}{k_0^2 - [\epsilon_{\mathbf{k}}^e(\phi_{n,h}^e)]^2}\;\left[G_0^\mp(K)\right]^{-1} \;. \tag{18}$$

The quasiparticle energies are

$$\epsilon_{\mathbf{k}}^e(\phi_{n,h}^e) \equiv \sqrt{(\mu - ek)^2 + |\phi_{n,h}^e|^2} \;, \tag{19}$$

where $\phi_{n,h}^e$ is the gap function for pairing of quarks ($e = +1$) or antiquarks ($e = -1$) with chirality h.

The right-hand side of (18) does not depend on the sign of the gap functions, *i.e.*, the difference in sign between $\phi_{2,h}^e$ and $\phi_{3,h}^e$, Eq. (13c), is irrelevant, $G_2^\pm \equiv G_3^\pm$. Then, Eq. (17) has the alternative representation

$$G^\pm(K) \equiv \mathbf{P_1}\,G_{\mathbf{1}}^\pm(K) + \mathbf{P_8}\,G_{\mathbf{8}}^\pm(K) \;, \tag{20}$$

where $\mathbf{P_{1,8}}$ are the singlet and octet projectors introduced by Zarembo [37], see above, and, following Eq. (16), $G_{\mathbf{1}}^\pm \equiv G_1^\pm$, $G_{\mathbf{8}}^\pm \equiv G_2^\pm = G_3^\pm$.

The off-diagonal components of the quasiparticle propagators are similarly computed as

$$\Xi^\pm(K) \equiv \sum_{n=1}^{3} \mathcal{C}^{(n)}\,\Xi_n^\pm(K) \;, \tag{21}$$

where

$$\Xi_n^+(K) = -\sum_{h=r,\ell}\sum_{e=\pm} \frac{\phi_{n,h}^e(K)}{k_0^2 - [\epsilon_{\mathbf{k}}^e(\phi_{n,h}^e)]^2}\,\mathcal{P}_{-h}\,\Lambda_{\mathbf{k}}^{-e} \quad , \quad \Xi_n^-(K) = -\sum_{h=r,\ell}\sum_{e=\pm} \frac{\left[\phi_{n,h}^e(K)\right]^*}{k_0^2 - [\epsilon_{\mathbf{k}}^e(\phi_{n,h}^e)]^2}\,\mathcal{P}_h\,\Lambda_{\mathbf{k}}^e \;. \tag{22}$$

Since $\phi_{2,h}^e = -\phi_{3,h}^e$, there is no simple representation in terms of singlet and octet projectors for $\Xi^\pm$. Nevertheless, in line with (16) let us define for future purpose $\Xi_{\mathbf{1}}^\pm \equiv \Xi_1^\pm$, $\Xi_{\mathbf{8}}^\pm \equiv \Xi_2^\pm \equiv -\Xi_3^\pm$.

Inserting Eqs. (17) and (21) into Eq. (8), one straightforwardly performs the trace over color and flavor space to obtain

$$
\begin{aligned}
\Pi^{\mu\nu}_{ab}(P) &= \delta_{ab}\,\Pi^{\mu\nu}(P) \;, \qquad &(23\text{a})\\
\Pi^{\mu\nu}(P) &= \frac{g^2}{12}\,\frac{T}{V}\sum_K \mathrm{Tr}_s\Big[\gamma^\mu\,G_{\mathbf{1}}^+(K)\,\gamma^\nu\,G_{\mathbf{8}}^+(K-P) + \gamma^\mu\,G_{\mathbf{8}}^+(K)\,\gamma^\nu\,G_{\mathbf{1}}^+(K-P)\\
&\quad + \gamma^\mu\,G_{\mathbf{1}}^-(K)\,\gamma^\nu\,G_{\mathbf{8}}^-(K-P) + \gamma^\mu\,G_{\mathbf{8}}^-(K)\,\gamma^\nu\,G_{\mathbf{1}}^-(K-P)\\
&\quad + 7\,\gamma^\mu\,G_{\mathbf{8}}^+(K)\,\gamma^\nu\,G_{\mathbf{8}}^+(K-P) + 7\,\gamma^\mu\,G_{\mathbf{8}}^-(K)\,\gamma^\nu\,G_{\mathbf{8}}^-(K-P)\\
&\quad + \gamma^\mu\,\Xi_{\mathbf{1}}^-(K)\,\gamma^\nu\,\Xi_{\mathbf{8}}^+(K-P) + \gamma^\mu\,\Xi_{\mathbf{8}}^-(K)\,\gamma^\nu\,\Xi_{\mathbf{1}}^+(K-P)\\
&\quad + \gamma^\mu\,\Xi_{\mathbf{1}}^+(K)\,\gamma^\nu\,\Xi_{\mathbf{8}}^-(K-P) + \gamma^\mu\,\Xi_{\mathbf{8}}^+(K)\,\gamma^\nu\,\Xi_{\mathbf{1}}^-(K-P)\\
&\quad + 2\,\gamma^\mu\,\Xi_{\mathbf{8}}^-(K)\,\gamma^\nu\,\Xi_{\mathbf{8}}^+(K-P) + 2\,\gamma^\mu\,\Xi_{\mathbf{8}}^+(K)\,\gamma^\nu\,\Xi_{\mathbf{8}}^-(K-P)\Big] \;. &(23\text{b})
\end{aligned}
$$

From (23) one learns two things. First, unlike the two-flavor case [19], the gluon self-energy is diagonal in the adjoint colors a,b. Second, there are no terms where both quasiparticle propagators involve singlet gaps. The reason is that such terms are proportional to $\mathrm{Tr}_c T_a\,\mathrm{Tr}_c T_b \equiv 0$.

The evaluation of the spin traces proceeds in complete analogy to the two-flavor case [19]. Assuming $\phi^e_{n,r} = -\phi^e_{n,\ell} \equiv \phi^e_n \in \mathbf{R}$, the result is [cf. Eq. (96a) of [19]]

$$
\begin{aligned}
\Pi^{\mu\nu}(P) = -\frac{g^2}{12}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\sum_{e_1,e_2=\pm}\Bigg(&\mathcal{T}_+^{\mu\nu}(\mathbf{k}_1,\mathbf{k}_2) \qquad (24)\\
\times\Bigg[&\left(\frac{\hat{n}_1\,(1-n_2)}{p_0+\hat{\epsilon}_1+\epsilon_2} - \frac{(1-\hat{n}_1)\,n_2}{p_0-\hat{\epsilon}_1-\epsilon_2}\right)(1-\hat{N}_1-N_2) + \left(\frac{(1-\hat{n}_1)\,(1-n_2)}{p_0-\hat{\epsilon}_1+\epsilon_2} - \frac{\hat{n}_1\,n_2}{p_0+\hat{\epsilon}_1-\epsilon_2}\right)(\hat{N}_1-N_2)\\
+&\left(\frac{n_1\,(1-\hat{n}_2)}{p_0+\epsilon_1+\hat{\epsilon}_2} - \frac{(1-n_1)\,\hat{n}_2}{p_0-\epsilon_1-\hat{\epsilon}_2}\right)(1-N_1-\hat{N}_2) + \left(\frac{(1-n_1)\,(1-\hat{n}_2)}{p_0-\epsilon_1+\hat{\epsilon}_2} - \frac{n_1\,\hat{n}_2}{p_0+\epsilon_1-\hat{\epsilon}_2}\right)(N_1-\hat{N}_2)\\
+7&\left(\frac{n_1\,(1-n_2)}{p_0+\epsilon_1+\epsilon_2} - \frac{(1-n_1)\,n_2}{p_0-\epsilon_1-\epsilon_2}\right)(1-N_1-N_2) + 7\left(\frac{(1-n_1)\,(1-n_2)}{p_0-\epsilon_1+\epsilon_2} - \frac{n_1\,n_2}{p_0+\epsilon_1-\epsilon_2}\right)(N_1-N_2)\Bigg]\\
&+\;\mathcal{T}_-^{\mu\nu}(\mathbf{k}_1,\mathbf{k}_2)\\
\times\Bigg[&\left(\frac{(1-\hat{n}_1)\,n_2)}{p_0+\hat{\epsilon}_1+\epsilon_2} - \frac{\hat{n}_1\,(1-n_2)}{p_0-\hat{\epsilon}_1-\epsilon_2}\right)(1-\hat{N}_1-N_2) + \left(\frac{\hat{n}_1\,n_2}{p_0-\hat{\epsilon}_1+\epsilon_2} - \frac{(1-\hat{n}_1)\,(1-n_2)}{p_0+\hat{\epsilon}_1-\epsilon_2}\right)(\hat{N}_1-N_2)\\
+&\left(\frac{(1-n_1)\,\hat{n}_2}{p_0+\epsilon_1+\hat{\epsilon}_2} - \frac{n_1\,(1-\hat{n}_2)}{p_0-\epsilon_1-\hat{\epsilon}_2}\right)(1-N_1-\hat{N}_2) + \left(\frac{n_1\,\hat{n}_2}{p_0-\epsilon_1+\hat{\epsilon}_2} - \frac{(1-n_1)\,(1-\hat{n}_2)}{p_0+\epsilon_1-\hat{\epsilon}_2}\right)(N_1-\hat{N}_2)\\
+7&\left(\frac{(1-n_1)\,n_2}{p_0+\epsilon_1+\epsilon_2} - \frac{n_1\,(1-n_2)}{p_0-\epsilon_1-\epsilon_2}\right)(1-N_1-N_2) + 7\left(\frac{n_1\,n_2}{p_0-\epsilon_1+\epsilon_2} - \frac{(1-n_1)\,(1-n_2)}{p_0+\epsilon_1-\epsilon_2}\right)(N_1-N_2)\Bigg]\\
&-\;\left[\mathcal{U}_+^{\mu\nu}(\mathbf{k}_1,\mathbf{k}_2)+\mathcal{U}_-^{\mu\nu}(\mathbf{k}_1,\mathbf{k}_2)\right]\\
\times\Bigg\{&\frac{\hat{\phi}_1\,\phi_2}{4\,\hat{\epsilon}_1\,\epsilon_2}\left[\left(\frac{1}{p_0+\hat{\epsilon}_1+\epsilon_2} - \frac{1}{p_0-\hat{\epsilon}_1-\epsilon_2}\right)(1-\hat{N}_1-N_2) - \left(\frac{1}{p_0-\hat{\epsilon}_1+\epsilon_2} - \frac{1}{p_0+\hat{\epsilon}_1-\epsilon_2}\right)(\hat{N}_1-N_2)\right]\\
+&\frac{\phi_1\,\hat{\phi}_2}{4\,\epsilon_1\,\hat{\epsilon}_2}\left[\left(\frac{1}{p_0+\epsilon_1+\hat{\epsilon}_2} - \frac{1}{p_0-\epsilon_1-\hat{\epsilon}_2}\right)(1-N_1-\hat{N}_2) - \left(\frac{1}{p_0-\epsilon_1+\hat{\epsilon}_2} - \frac{1}{p_0+\epsilon_1-\hat{\epsilon}_2}\right)(N_1-\hat{N}_2)\right]\\
+2&\frac{\phi_1\,\phi_2}{4\,\epsilon_1\,\epsilon_2}\left[\left(\frac{1}{p_0+\epsilon_1+\epsilon_2} - \frac{1}{p_0-\epsilon_1-\epsilon_2}\right)(1-N_1-N_2) - \left(\frac{1}{p_0-\epsilon_1+\epsilon_2} - \frac{1}{p_0+\epsilon_1-\epsilon_2}\right)(N_1-N_2)\right]\Bigg\}\Bigg)\,.
\end{aligned}
$$

Here, I denoted the octet and singlet gaps by

$$
\phi_i \equiv \phi_{\mathbf{8}}^{e_i} \quad , \quad \hat{\phi}_i \equiv \phi_{\mathbf{1}}^{e_i} \;. \qquad (25)
$$

Correspondingly,

$$\epsilon_i \equiv \epsilon_{\mathbf{k}_i}^{e_i}(\phi_i) \quad , \quad \hat{\epsilon}_i \equiv \epsilon_{\mathbf{k}_i}^{e_i}(\hat{\phi}_i) \tag{26}$$

are the excitation energies for quasiparticles with octet and singlet gaps, $\mathbf{k}_1 \equiv \mathbf{k}$, $\mathbf{k}_2 \equiv \mathbf{k} - \mathbf{p}$,

$$n_i \equiv n_{\mathbf{k}_i}^{e_i} \equiv \frac{\epsilon_i - \xi_i}{2\,\epsilon_i} \quad , \quad \hat{n}_i \equiv \hat{n}_{\mathbf{k}_i}^{e_i} \equiv \frac{\hat{\epsilon}_i - \xi_i}{2\,\hat{\epsilon}_i} \tag{27}$$

are the occupation numbers for quasiparticles with octet and singlet gaps, $\xi_i \equiv e_i\, k_i - \mu$, and

$$N_i \equiv N_{\mathbf{k}_i}^{e_i} \equiv \left[\exp\left(\frac{\epsilon_i}{T}\right) + 1\right]^{-1} \quad , \quad \hat{N}_i \equiv \hat{N}_{\mathbf{k}_i}^{e_i} \equiv \left[\exp\left(\frac{\hat{\epsilon}_i}{T}\right) + 1\right]^{-1} \tag{28}$$

are the corresponding thermal occupation numbers. The spin traces are [see Eqs. (45) and (97) of [19]]

$$\mathcal{T}_\pm^{00} = \mathcal{U}_\pm^{00} = 1 + e_1 e_2\, \hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_2 \ , \tag{29a}$$

$$\mathcal{T}_\pm^{0i} = \mathcal{T}_\pm^{i0} = -\mathcal{U}_\pm^{0i} = \mathcal{U}_\pm^{i0} = \pm e_1\, \hat{k}_1^i \pm e_2\, \hat{k}_2^i \quad , \quad i = x, y, z \ , \tag{29b}$$

$$\mathcal{T}_\pm^{ij} = -\mathcal{U}_\pm^{ij} = \delta^{ij} \left(1 - e_1 e_2\, \hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_2\right) + e_1 e_2 \left(\hat{k}_1^i\, \hat{k}_2^j + \hat{k}_1^j\, \hat{k}_2^i\right) \quad , \quad i, j = x, y, z \ . \tag{29c}$$

In Eq. (24), the terms proportional to $\mathcal{T}_\pm^{\mu\nu}$ correspond to the diagram in Fig. 1(a), while those proportional to $\mathcal{U}_\pm^{\mu\nu}$ correspond to that in Fig. 1(b).

In order to compute the Debye and Meissner mass, it is sufficient to consider the time-like, $\mu = \nu = 0$, and space-like, $\mu = i$, $\nu = j$, components of the self-energy, since the Debye and Meissner masses are defined as

$$m_D^2 \equiv - \lim_{p \to 0} \Pi^{00}(0, p) \quad , \quad m_M^2 \equiv \lim_{p \to 0} \Pi^{ii}(0, p) \ . \tag{30}$$

The sign in the first equation is due to the choice of metric. The self-energy of electric gluons is

$$\begin{aligned}
\Pi^{00}(P) = -\frac{g^2}{12} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \sum_{e_1, e_2 = \pm} & (1 + e_1 e_2\, \hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_2) \\
\times \Bigg[& \left(\frac{1}{p_0 + \hat{\epsilon}_1 + \epsilon_2} - \frac{1}{p_0 - \hat{\epsilon}_1 - \epsilon_2}\right) (1 - \hat{N}_1 - N_2)\, \frac{\hat{\epsilon}_1\, \epsilon_2 - \xi_1\, \xi_2 - \hat{\phi}_1\, \phi_2}{2\, \hat{\epsilon}_1\, \epsilon_2} \\
+ & \left(\frac{1}{p_0 + \epsilon_1 + \hat{\epsilon}_2} - \frac{1}{p_0 - \epsilon_1 - \hat{\epsilon}_2}\right) (1 - N_1 - \hat{N}_2)\, \frac{\epsilon_1\, \hat{\epsilon}_2 - \xi_1\, \xi_2 - \phi_1\, \hat{\phi}_2}{2\, \epsilon_1\, \hat{\epsilon}_2} \\
+7 & \left(\frac{1}{p_0 + \epsilon_1 + \epsilon_2} - \frac{1}{p_0 - \epsilon_1 - \epsilon_2}\right) (1 - N_1 - N_2)\, \frac{\epsilon_1\, \epsilon_2 - \xi_1\, \xi_2 - 2\, \phi_1\, \phi_2/7}{2\, \epsilon_1\, \epsilon_2} \\
+ & \left(\frac{1}{p_0 - \hat{\epsilon}_1 + \epsilon_2} - \frac{1}{p_0 + \hat{\epsilon}_1 - \epsilon_2}\right) (\hat{N}_1 - N_2)\, \frac{\hat{\epsilon}_1\, \epsilon_2 + \xi_1\, \xi_2 + \hat{\phi}_1\, \phi_2}{2\, \hat{\epsilon}_1\, \epsilon_2} \\
+ & \left(\frac{1}{p_0 - \epsilon_1 + \hat{\epsilon}_2} - \frac{1}{p_0 + \epsilon_1 - \hat{\epsilon}_2}\right) (N_1 - \hat{N}_2)\, \frac{\epsilon_1\, \hat{\epsilon}_2 + \xi_1\, \xi_2 + \phi_1\, \hat{\phi}_2}{2\, \epsilon_1\, \hat{\epsilon}_2} \\
+7 & \left(\frac{1}{p_0 - \epsilon_1 + \epsilon_2} - \frac{1}{p_0 + \epsilon_1 - \epsilon_2}\right) (N_1 - N_2)\, \frac{\epsilon_1\, \epsilon_2 + \xi_1\, \xi_2 + 2\, \phi_1\, \phi_2/7}{2\, \epsilon_1\, \epsilon_2} \Bigg] \ .
\end{aligned} \tag{31a}$$

On the other hand, the self-energy of magnetic gluons is

$$\begin{aligned}
\Pi^{ij}(P) = -\frac{g^2}{12} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \sum_{e_1, e_2 = \pm} & \left[\delta^{ij} \left(1 - e_1 e_2\, \hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_2\right) + e_1 e_2 \left(\hat{k}_1^i\, \hat{k}_2^j + \hat{k}_1^j\, \hat{k}_2^i\right)\right] \\
\times \Bigg[& \left(\frac{1}{p_0 + \hat{\epsilon}_1 + \epsilon_2} - \frac{1}{p_0 - \hat{\epsilon}_1 - \epsilon_2}\right) (1 - \hat{N}_1 - N_2)\, \frac{\hat{\epsilon}_1\, \epsilon_2 - \xi_1\, \xi_2 + \hat{\phi}_1\, \phi_2}{2\, \hat{\epsilon}_1\, \epsilon_2} \\
+ & \left(\frac{1}{p_0 + \epsilon_1 + \hat{\epsilon}_2} - \frac{1}{p_0 - \epsilon_1 - \hat{\epsilon}_2}\right) (1 - N_1 - \hat{N}_2)\, \frac{\epsilon_1\, \hat{\epsilon}_2 - \xi_1\, \xi_2 + \phi_1\, \hat{\phi}_2}{2\, \epsilon_1\, \hat{\epsilon}_2}
\end{aligned}$$

$$+7\left(\frac{1}{p_0+\epsilon_1+\epsilon_2}-\frac{1}{p_0-\epsilon_1-\epsilon_2}\right)(1-N_1-N_2)\,\frac{\epsilon_1\,\epsilon_2-\xi_1\,\xi_2+2\,\phi_1\,\phi_2/7}{2\,\epsilon_1\,\epsilon_2}$$
$$+\left(\frac{1}{p_0-\hat\epsilon_1+\epsilon_2}-\frac{1}{p_0+\hat\epsilon_1-\epsilon_2}\right)(\hat N_1-N_2)\,\frac{\hat\epsilon_1\,\epsilon_2+\xi_1\,\xi_2-\hat\phi_1\,\phi_2}{2\,\hat\epsilon_1\,\epsilon_2}$$
$$+\left(\frac{1}{p_0-\epsilon_1+\hat\epsilon_2}-\frac{1}{p_0+\epsilon_1-\hat\epsilon_2}\right)(N_1-\hat N_2)\,\frac{\epsilon_1\,\hat\epsilon_2+\xi_1\,\xi_2-\phi_1\,\hat\phi_2}{2\,\epsilon_1\,\hat\epsilon_2}$$
$$\left.+7\left(\frac{1}{p_0-\epsilon_1+\epsilon_2}-\frac{1}{p_0+\epsilon_1-\epsilon_2}\right)(N_1-N_2)\,\frac{\epsilon_1\,\epsilon_2+\xi_1\,\xi_2-2\,\phi_1\,\phi_2/7}{2\,\epsilon_1\,\epsilon_2}\right] \,. \tag{31b}$$

In Eq. (31), terms proportional to $(\epsilon_1\,\epsilon_2 \pm \xi_1\,\xi_2)/(2\,\epsilon_1\,\epsilon_2)$ (and similar terms involving the singlet gaps, $\hat\phi_i$) arise from the diagram in Fig. 1(a), while terms proportional to $\phi_1\,\phi_2/(2\,\epsilon_1\,\epsilon_2)$ (and similar terms involving the singlet gaps, $\hat\phi_i$) arise from that in Fig. 1(b).

To proceed I treat quasi-antiparticles as free antiparticles, as in [19],

$$\phi_{\mathbf 8}^- \simeq 0 \quad , \quad \phi_{\mathbf 1}^- \simeq 0 \quad , \quad \epsilon_{\mathbf k_i}^-(\phi_{\mathbf 8}^-) \simeq k_i+\mu \quad , \quad \epsilon_{\mathbf k_i}^-(\phi_{\mathbf 1}^-) \simeq k_i+\mu \,, \tag{32a}$$
$$n_{\mathbf k_i}^- \simeq 1 \quad , \quad \hat n_{\mathbf k_i}^- \simeq 1 \quad , \quad 1-n_{\mathbf k_i}^- \simeq 0 \quad , \quad 1-\hat n_{\mathbf k_i}^- \simeq 0 \quad , \quad N_{\mathbf k_i}^- \simeq 0 \quad , \quad \hat N_{\mathbf k_i}^- \simeq 0 \,. \tag{32b}$$

Setting $p_0=0$ and sending $p\to 0$, there are then no antiparticle contributions to the electric part of the gluon self-energy. Abbreviating $\phi_{\mathbf 8}^+\equiv\phi$, $\phi_{\mathbf 1}^+\equiv\hat\phi$, $\epsilon_{\mathbf k}^+\equiv\epsilon$, $\hat\epsilon_{\mathbf k}^+\equiv\hat\epsilon$, $N_{\mathbf k}^+\equiv N$, $\hat N_{\mathbf k}^+\equiv\hat N$,

$$\Pi^{00}(0)\equiv \Pi_{\rm e}^{(a)}(0)+\Pi_{\rm e}^{(b)}(0)\,, \tag{33a}$$
$$\Pi_{\rm e}^{(a)}(0)\simeq -\frac{g^2}{24\pi^2}\int_0^\infty dk\,k^2\left[8\,\frac{1-\hat N-N}{\hat\epsilon+\epsilon}\,\frac{\hat\epsilon\,\epsilon-\xi^2}{2\,\hat\epsilon\,\epsilon}+7\,(1-2\,N)\,\frac{\phi^2}{\epsilon^3}\right.$$
$$\left.-\,8\,\frac{N-\hat N}{\epsilon-\hat\epsilon}\,\frac{\hat\epsilon\,\epsilon+\xi^2}{2\,\hat\epsilon\,\epsilon}-28\,\frac{{\rm d}N}{{\rm d}\epsilon}\left(1-\frac{\phi^2}{2\,\epsilon^2}\right)\right]\,, \tag{33b}$$
$$\Pi_{\rm e}^{(b)}(0)\simeq -\frac{g^2}{24\pi^2}\int_0^\infty dk\,k^2\left[-8\,\frac{1-\hat N-N}{\hat\epsilon+\epsilon}\,\frac{\hat\phi\,\phi}{2\,\hat\epsilon\,\epsilon}-2\,(1-2\,N)\,\frac{\phi^2}{\epsilon^3}\right.$$
$$\left.-8\,\frac{N-\hat N}{\epsilon-\hat\epsilon}\,\frac{\hat\phi\,\phi}{2\,\hat\epsilon\,\epsilon}-4\,\frac{{\rm d}N}{{\rm d}\epsilon}\,\frac{\phi^2}{\epsilon^2}\right]\,. \tag{33c}$$

In order to facilitate comparison with the results of Son and Stephanov, and to elucidate the origin of the various terms, I have separated the self-energy into the contributions from the diagram in Fig. 1(a), $\Pi_{\rm e}^{(a)}$, and that in Fig. 1(b), $\Pi_{\rm e}^{(b)}$.

For the self-energy of magnetic gluons one obtains with $\int d\Omega\,\hat k^i\,\hat k^j/(4\pi)=\delta^{ij}/3$, $n_{\mathbf k}^+\equiv n$, $\hat n_{\mathbf k}^+\equiv\hat n$,

$$\Pi^{ij}(0)\equiv\delta^{ij}\left[\Pi_{\rm m}^{(a1)}(0)+\Pi_{\rm m}^{(a2)}(0)+\Pi_{\rm m}^{(b)}(0)\right]\,, \tag{33d}$$
$$\Pi_{\rm m}^{(a1)}(0)\simeq -\frac{g^2}{72\pi^2}\int_0^\infty dk\,k^2\left[8\,\frac{1-\hat N-N}{\hat\epsilon+\epsilon}\,\frac{\hat\epsilon\,\epsilon-\xi^2}{2\,\hat\epsilon\,\epsilon}+7\,(1-2\,N)\,\frac{\phi^2}{\epsilon^3}\right.$$
$$\left.-\,8\,\frac{N-\hat N}{\epsilon-\hat\epsilon}\,\frac{\hat\epsilon\,\epsilon+\xi^2}{2\,\hat\epsilon\,\epsilon}-28\,\frac{{\rm d}N}{{\rm d}\epsilon}\left(1-\frac{\phi^2}{2\,\epsilon^2}\right)\right]\,, \tag{33e}$$
$$\Pi_{\rm m}^{(a2)}(0)\simeq -\frac{g^2}{72\pi^2}\int_0^\infty dk\,k^2\left[16\,\frac{(1-\hat N)(1-\hat n)}{k+\mu+\hat\epsilon}+128\,\frac{(1-N)(1-n)}{k+\mu+\epsilon}\right.$$
$$\left.+16\,\frac{\hat N\,\hat n}{k+\mu-\hat\epsilon}+128\,\frac{N\,n}{k+\mu-\epsilon}-72\,\frac{1}{k}\right]\,, \tag{33f}$$
$$\Pi_{\rm m}^{(b)}(0)\simeq -\frac{g^2}{72\pi^2}\int_0^\infty dk\,k^2\left[8\,\frac{1-\hat N-N}{\hat\epsilon+\epsilon}\,\frac{\hat\phi\,\phi}{2\,\hat\epsilon\,\epsilon}+2\,(1-2\,N)\,\frac{\phi^2}{\epsilon^3}\right.$$

$$+\,8\,\frac{N-\hat{N}}{\epsilon-\hat{\epsilon}}\,\frac{\hat{\phi}\,\phi}{2\,\hat{\epsilon}\,\epsilon}+4\,\frac{\mathrm{d}N}{\mathrm{d}\epsilon}\,\frac{\phi^2}{\epsilon^2}\Bigg]\ . \tag{33g}$$

Again, I have separated contributions from Fig. 1(a), $\Pi_{\rm m}^{(a1)}+\Pi_{\rm m}^{(a2)}$, from those of Fig. 1(b), $\Pi_{\rm m}^{(b)}$. Obviously, $\Pi_{\rm e}^{(a)}\equiv 3\,\Pi_{\rm m}^{(a1)}$, $\Pi_{\rm e}^{(b)}\equiv -3\,\Pi_{\rm m}^{(b)}$. There are two contributions from Fig. 1(a). The first, $\Pi_{\rm m}^{(a1)}$, arises from quasiparticle-quasiparticle excitations, while the second, $\Pi_{\rm m}^{(a2)}$, originates from quasiparticle-antiparticle excitations. The latter is UV-divergent, and thus requires renormalization, which is achieved by adding the last term in Eq. (33f). As we shall see shortly, $\Pi_{\rm m}^{(a2)}$ gives rise to the "bare" Meissner mass in Hong's effective theory [21,26], where contributions involving antiparticles are integrated out.

Let us now consider the case of zero temperature, where $N\equiv\hat{N}\equiv 0$. One may restrict the k integration to the region $0\leq k\leq 2\mu$, the contribution from $k\geq 2\mu$ can be shown to be negligible. The various components of the self-energies are

$$\Pi_{\rm e}^{(a)}(0)\equiv 3\,\Pi_{\rm m}^{(a1)}(0)\simeq -\frac{g^2\mu^2}{12\pi^2}\int_0^\mu \mathrm{d}\xi\,\left[\frac{4}{\hat{\phi}^2-\phi^2}\,\left(\frac{\hat{\epsilon}^2+\xi^2}{\hat{\epsilon}}-\frac{\epsilon^2+\xi^2}{\epsilon}\right)+7\,\frac{\phi^2}{\epsilon^3}\right]\ , \tag{34a}$$

$$\Pi_{\rm e}^{(b)}(0)\equiv -3\,\Pi_{\rm m}^{(b)}(0)\simeq -\frac{g^2\mu^2}{12\pi^2}\int_0^\mu \mathrm{d}\xi\,\left[-\frac{4\,\hat{\phi}\,\phi}{\hat{\phi}^2-\phi^2}\,\left(\frac{1}{\epsilon}-\frac{1}{\hat{\epsilon}}\right)-2\,\frac{\phi^2}{\epsilon^3}\right]\ , \tag{34b}$$

$$\Pi_{\rm m}^{(a2)}(0)\simeq\frac{g^2}{72\pi^2}\int_0^{2\mu}\mathrm{d}k\,k\,\left[8\,\frac{\mu\,(\hat{\epsilon}-k+\mu)+\hat{\phi}^2}{\hat{\epsilon}\,(\hat{\epsilon}+k+\mu)}+64\,\frac{\mu\,(\epsilon-k+\mu)+\phi^2}{\epsilon\,(\epsilon+k+\mu)}\right]\ . \tag{34c}$$

The integral appearing in (34c) was already computed in [19], Eq. (122). The result is

$$\Pi_{\rm m}^{(a2)}(0)\simeq m_g^2\ . \tag{35}$$

As advertised above, this is the "bare" Meissner mass appearing in Hong's effective theory [21,26].

To obtain the expressions (34a) and (34b) for $\Pi_{\rm e}^{(a)}$ and $\Pi_{\rm e}^{(b)}$, I substituted $\xi\equiv k-\mu$ and exploited the symmetry of the integrand around $\xi=0$. Furthermore, contributions $\sim\xi^2/\mu^2$ in the integrands were neglected, because they give rise to terms of order $\sim\phi^2/\mu^2$ relative to the leading terms. Neglecting the momentum dependence of the gap function, all remaining integrals are exactly solvable. First note that, to leading order, $\int_0^\mu \mathrm{d}\xi\,\phi^2/\epsilon^3\simeq 1$. This takes care of the last term in Eqs. (34a) and (34b). To compute the first, substitute $y\equiv\ln[(\xi+\epsilon)/\phi]$ for ξ in the first term in parentheses, and $\hat{y}\equiv\ln[(\xi+\hat{\epsilon})/\hat{\phi}]$ in the second. Evaluating the y integral, note that one must not approximate $\ln[(\mu+\sqrt{\mu^2+\phi^2})/\phi]\simeq\ln(2\mu/\phi)$ for the upper boundary of the y integral, and similarly for the $\hat{y}$ integral. The reason is that leading-order terms cancel between these two integrals. The subleading terms conspire to cancel the denominator $\hat{\phi}^2-\phi^2$. To leading order, one then obtains

$$\Pi_{\rm e}^{(a)}(0)\simeq -\frac{3}{2}\,m_g^2\quad,\quad \Pi_{\rm e}^{(b)}(0)\simeq\frac{1}{3}\,m_g^2\,\left(1+\frac{2\,\hat{\phi}\,\phi}{\hat{\phi}^2-\phi^2}\,\ln\frac{\hat{\phi}}{\phi}\right)\ . \tag{36}$$

Neglecting the sextet gap, the singlet gap is twice the octet gap, $\hat{\phi}=2\phi$, cf. Eq. (13), and

$$\Pi_{\rm e}^{(b)}(0)\simeq\frac{1}{3}\,m_g^2\,\left(1+\frac{4}{3}\,\ln 2\right)\ . \tag{37}$$

Using the definitions of the Debye and Meissner masses, Eq. (30), this confirms the results of Son and Stephanov, Eq. (5), *q.e.d.*

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